Financial markets dependence modeling using vine copulae

Vine copulae estimation of asset decomposition

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Table of content

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Context of the project

- Luxembourg is the 10th biggest insurance hub in Europe with a total assets balance sheet of over 227Bn EUR [0], which is composed mainly of financial assets to cover insurance premiums.
- The project is supported by the private sector for better validation in control for financial institution familiar with asset management.

[0] pwc Luxembourg

Characteristics of insurance industry

Focus on insurances with guaranteed capital (equivalence of saving accounts with fiscal advantages for the beneficiaries in case of death of the insured person)

- Tendency to have a lot of assets (500 to 5000 assets), especially for insurances with capital guaranteed
- Very low transparency in portfolio composition
- Highly diversified and usually considered as very low risk investments

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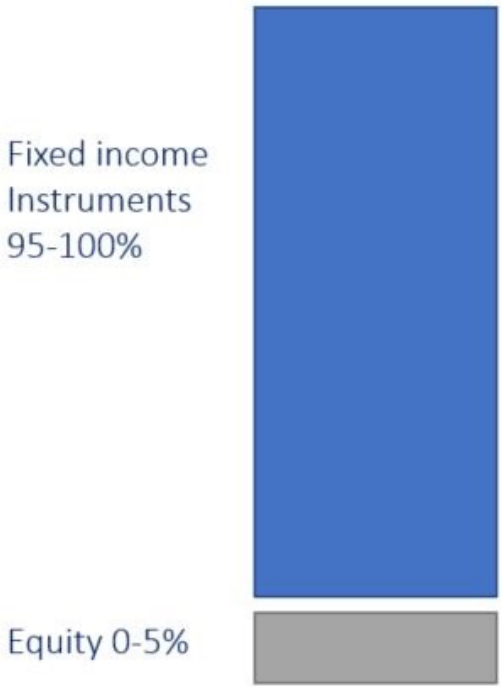
Typical insurance portfolio composition



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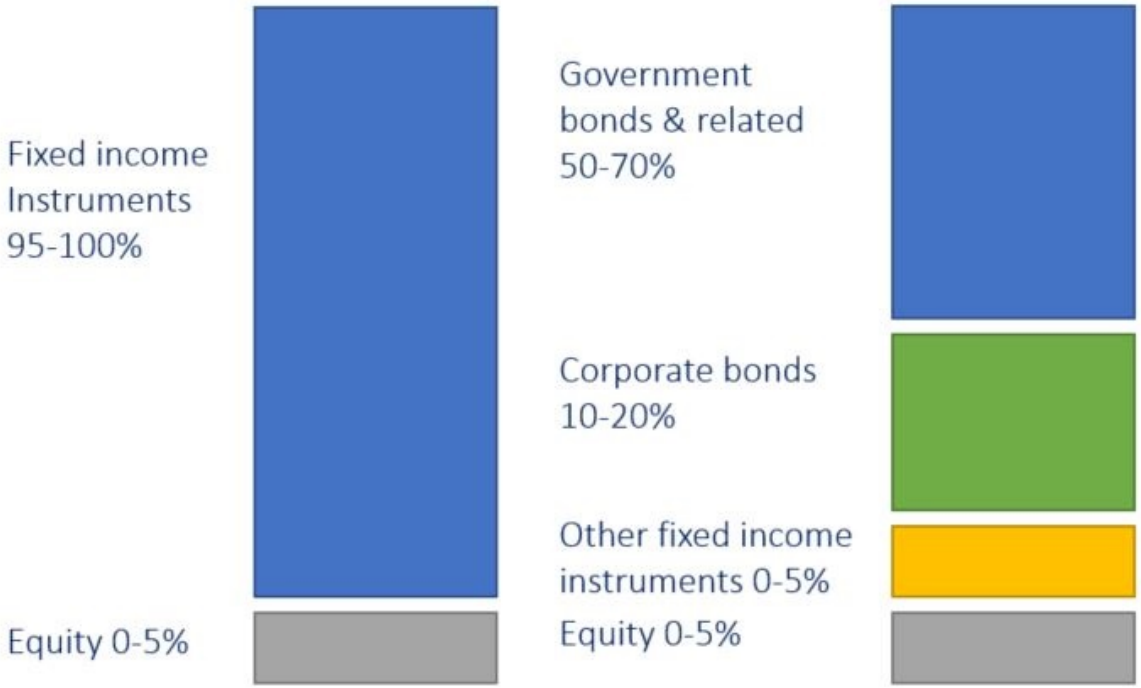

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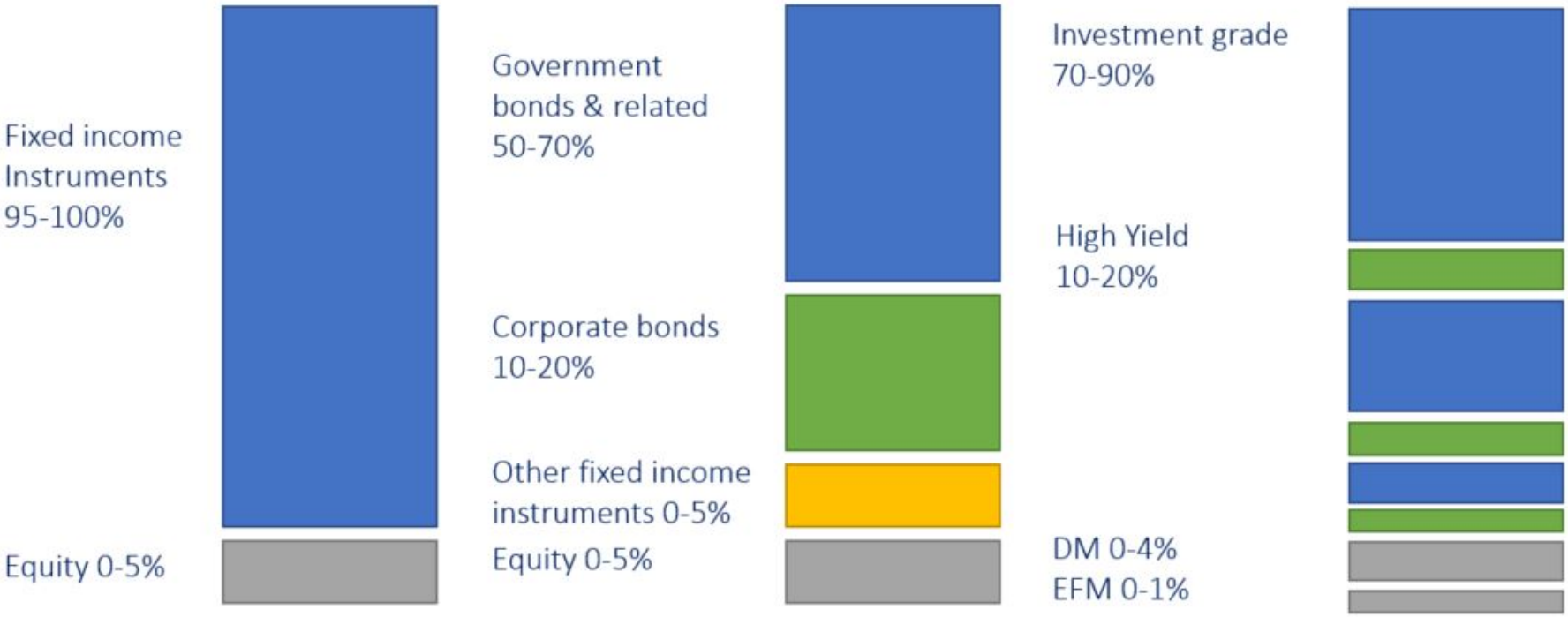

Table of content

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Definition & Providers

- A market index is a selection of a pool of assets selected according to some crietria.
- Main indices providers are MSCI, Markit, ICE, HFR, Bloomberg, FTSE, JPM, ...

IHS Markit™

MSCI 

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Decomposition methodology: leverage on indices characteristics

Main characteristics of equity

- Index category: country, country sectorial, country style, region, region sectorial, region style
- Sector or Industry: Communications, Energy, Materials...
- Investment styles: size (small, mid, big cap), growth, value
- Currency exposure (local currency of constituents)
- Region (World, NA, UE, Asia) or market (DM, EM, FT)

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Decomposition methodology: leverage on indices characteristics

Main characteristics of fixed income

- Credit rating: investment grade or high yield
- Maturity or maturity range: 1-3Y, 3-5Y, 5-10Y, 10Y+
- Bond type: corporate, government, convertible, inflation linked, high yield, investment grade
- Currency exposure (local currency of constituents)
- Region (World, NA, UE, Asia), market (DM, EM, FT) or country

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Table of content

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Copulae

Definition

$C : [0,1]^d \to [0,1]$ is a d-dimensional copula function if C is a joint cumulative distribution function of a d-dimensional random vector on the unit cube with uniform marginals. .

Theorem (Sklar, 1959)

For continuous random variables $(X_1, ..., X_d)$ of cdf H, the copula function C, given by

$$C(F_1(x_1), ..., F_d(x_d)) = H(x_1, ..., x_d)$$

is unique.

If h denotes the density of the multivariate distribution, and c the copula density from the cdf C,

$$c(F_1(x_1), ..., F_d(x_d)).f_1(x_1)...f_d(x_d) = h(x_1, ..., x_d).$$

Use of multivariate densities

There exists extensive research for the bi-variate case for which a lot of different families have been found (Nelsen, 2006).

These models naturally extend to multivariate densities, especially to the case with pair decomposition (Bedford & Cooke, 2001), (Bedford & Cooke, 2001). We have

$$f(x_1, ..., x_d) = f(x_d)f(x_{d-1}|x_d)...f(x_1|x_2, ..., x_d), \tag{1}$$

where

$$f(x_{d-1}|x_d) = c_{d,d-1}(F_d(x_d), F_{d-1}(x_{d-1})).f_d(x_d). \tag{2}$$

R-Vine copula construction

The variable decomposition has been organized by means of trees composed of pair variables e.g. R-Vine copula (Dissman et al, 2013).

- The tree T_1 has nodes $N_1 = 1, ..., d$ and edges E_1.
- For $i = 2, ..., d-1$, the nodes of T_i are the edges of T_{i-1},i.e., $N_i = E_{i-1}$.
- Proximity condition: if two nodes are connected by an edge on tree T_{i+1}, the nodes (which are edges on tree T_i) are connected by the same node on the tree T_i.

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Table of content

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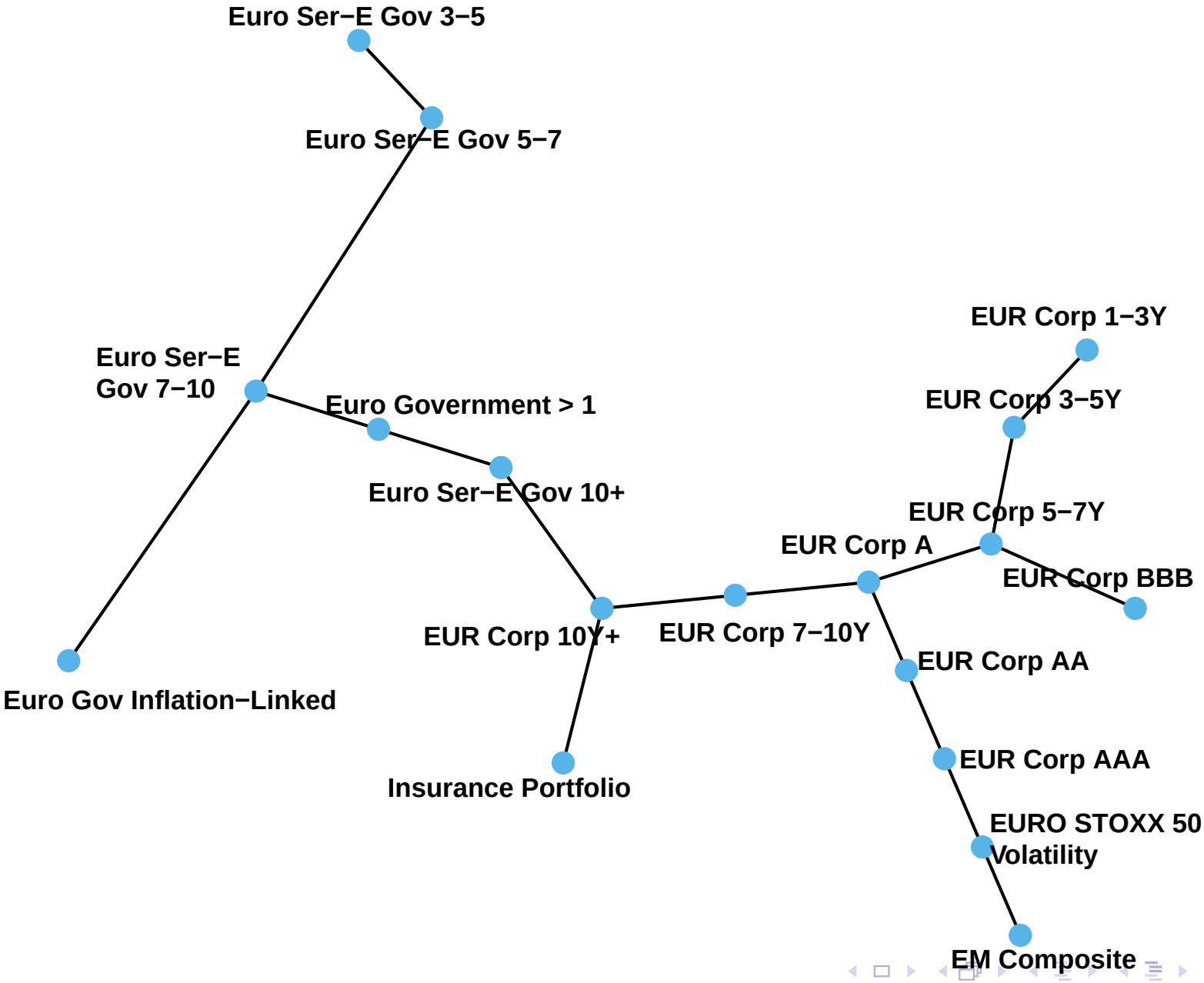
Insurance portfolio asset decomposition

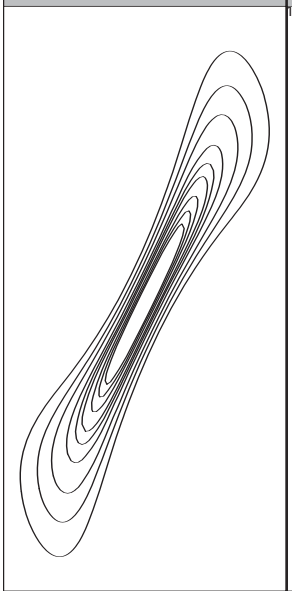
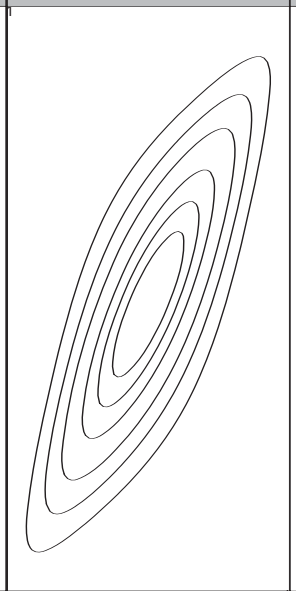
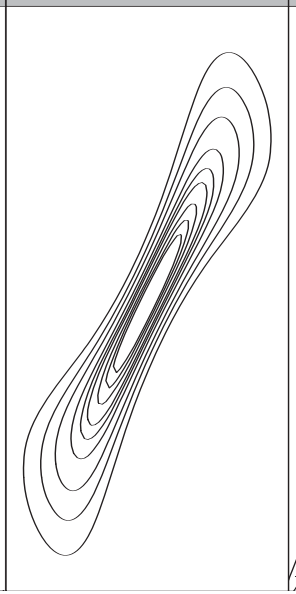
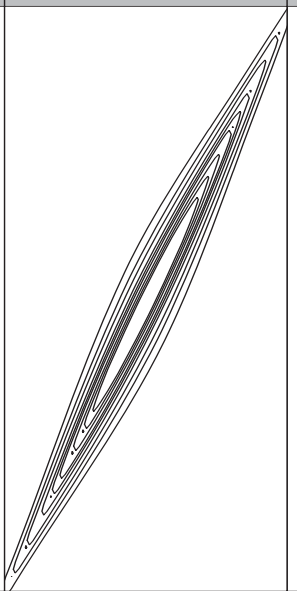
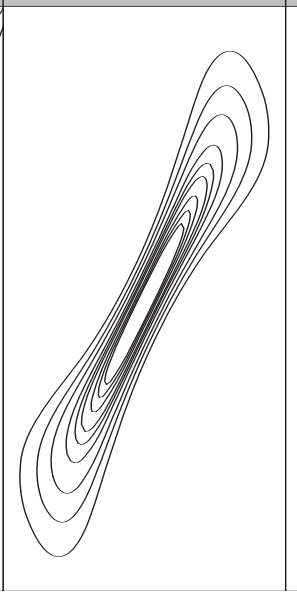
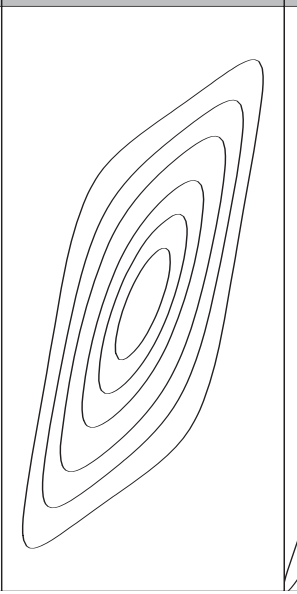
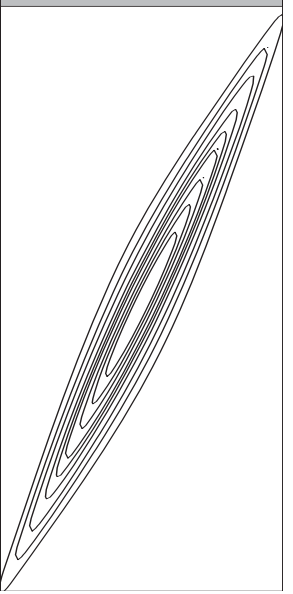
The target is

- Financial portfolio of more than 1000 assets with a fixed income predominance
- Period from January 2011 to April 2019
- Main indices selected depending on main insurance portfolios characteristics

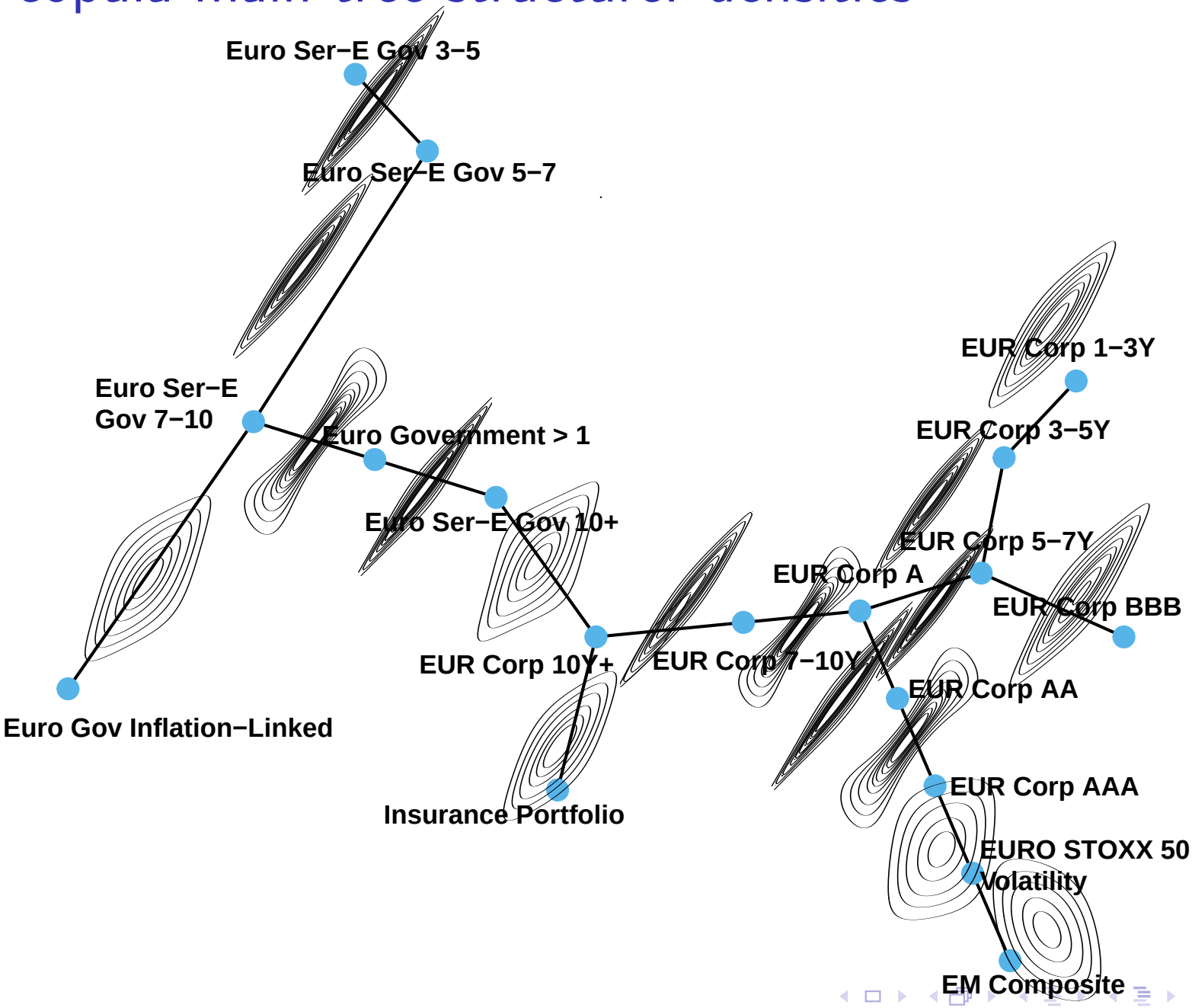
Vine copula main tree structure



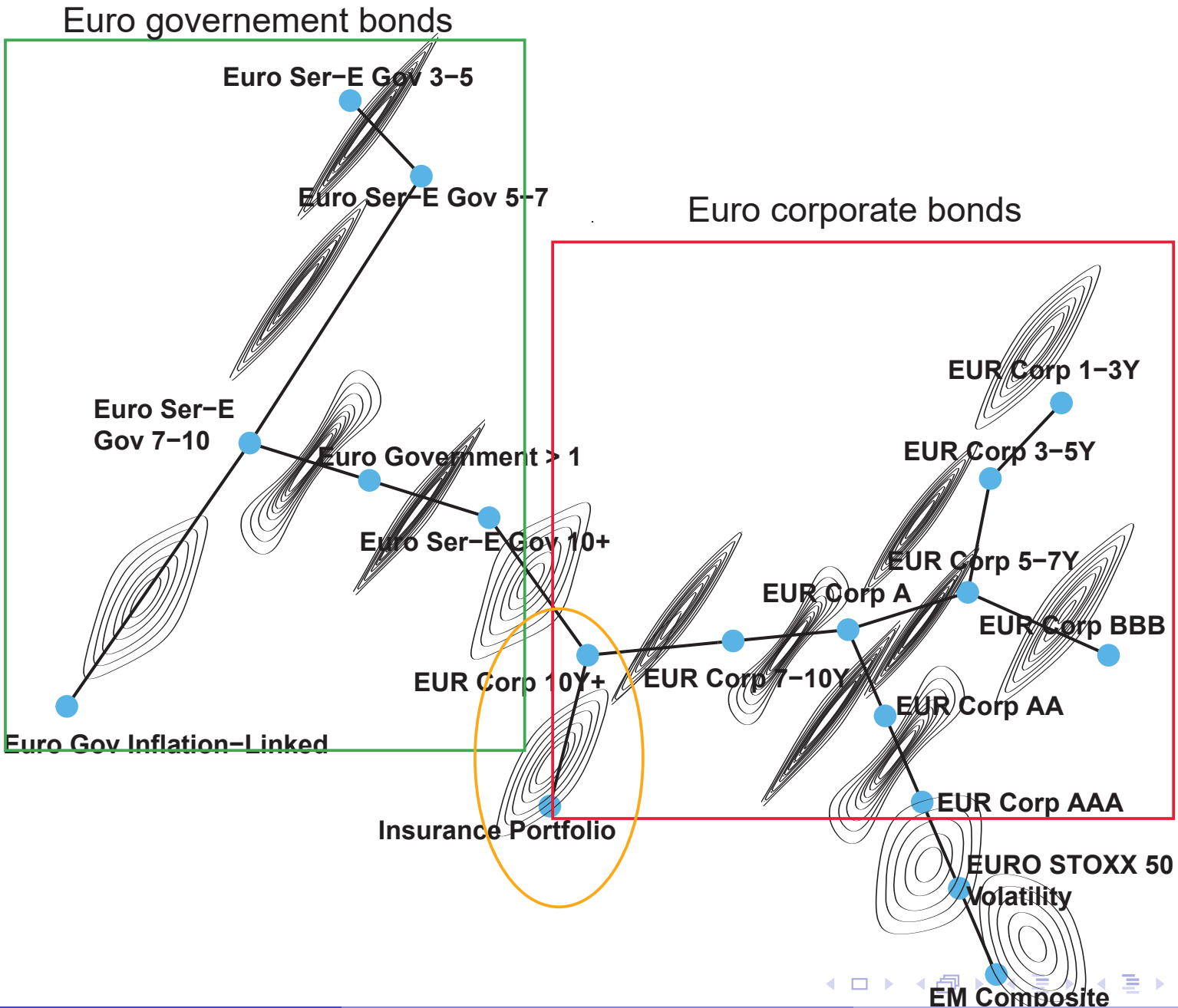
Main dependencies

Euro Ser−E Gov 7-10, Euro Government >1	Insurance Portfolio, EUR Corp 10Y+	EUR Corp AAA, EUR Corp AA	EUR Corp AA, EUR Corp A	EUR Corp A, EUR Corp 7-10Y	Euro Ser−E, Gov 10+, EUR Corp 10Y+	EUR Corp 10Y +, EUR Corp 7-10Y
						

Vine copula main tree structure: densities



Vine copula main structure: asset decomposition



Conclusions

- Vine copulas are very flexible to understand underlying portfolio risks
- The model allows to compute quantitative values of the risks
- Insurances portfolios offer good results with financial indices due to high diversification

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